Two-component spreading phenomena: Why the geometry makes the criticality

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We have numerically and theoretically investigated a simple model for two-component spreading phenomena in two different growth geometries (i.e., spreading confined in a half space and spreading in a free space). The criticality of the domain substructures unexpectedly depends on the considered geometry. This is understood by simple arguments of domain-wall particle diffusion and annihilation. We derive a relationship between the critical exponents χ and α for domain-wall spatial distributions in different geometries. The latter relationship is numerically verified in two, three, and four dimensions. [S1063-651X(96)10209-9]

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Spreading and invading phenomena are common nonequilibrium processes in nature as liquid invasion in porous medium, grain coalescence in alloys, fracture propagation in solids, damage spreading in electrical or neural networks, and virus propagation. For spreading processes driven by cooperative or nonlinear evolution rules, the systems develop patterns that often reach a high level of complexity [1].

The most simple spreading model is the Eden model, which describes the aggregation of identical particles. The model was imagined in order to mimic the growth of bacteria cell colonies [2] and was rapidly generalized to simulate other one-component spreading phenomena [3,4]. In version C of the Eden model, [4] a single step of the growth consists of randomly selecting a particle on the surface of a seed, a cluster thereafter, and at random filling one of its empty neighbors by a new particle. Different geometries have been studied: the seed was chosen to be, e.g., an occupied line or a single occupied site. In all cases, the generated clusters are found to fill the entire available space showing *trivial* nonfractal structures.

However, in natural systems, the growing entities usually present additional degrees of freedom. Examples of multicomponent systems are alloys, fluids, magnets, ceramics, polymers, bacterian cells, and viruses. The pattern formation in multicomponent systems is then of great interest [5]. The aim of this paper is to investigate the pattern formation of multicomponent spreadings in different geometries. We will restrict our discussion to two-component cases, which is obviously the first major step in elaborating a general framework for multicomponent growths.

Recently, Saito and Müller-Krumbhaar [6] generalized the Eden model on a square lattice in considering the competition of two different species A and B. Starting on *a seed line* of antiferromagneticlike configuration $ABAB\ldots$, the two-dimensional growth takes place by selecting randomly, at each growth step, some particle A (or B) on a cluster surface. The (A or B) selected particle gives an offspring *of the same species* on a randomly chosen empty neighboring site of the selected particle site. The macroscopic growth takes place in a direction perpendicular to the seed line. Periodic boundary conditions are used in this confined half space. Note that this basic model excludes the nucleation of a B species in a neighborhood consisting only of A species and vice versa. The Saito–Müller-Krumbhaar (SMK) growth rule leads to the formation of domains that compete for growth. Some domains are slowly trapped by others that coalesce. The process is far from equilibrium and history dependent such that the internal pattern reflects the growth itself.

Due to the coalescence of domains, the competition between domains leads asymptotically to one domain winning the competition and covering the whole cluster surface as illustrated in Fig. 1. For $d=1+1$, the number $n(h)$ of A or

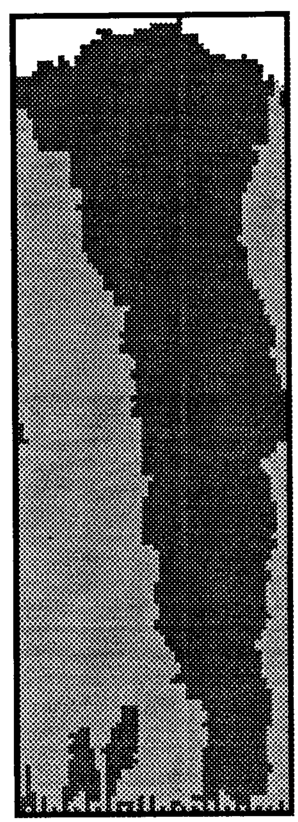

FIG. 1. Internal structure of a cluster grown with the SMK rule (in a half space of size $L=64$). Each color represents a species.

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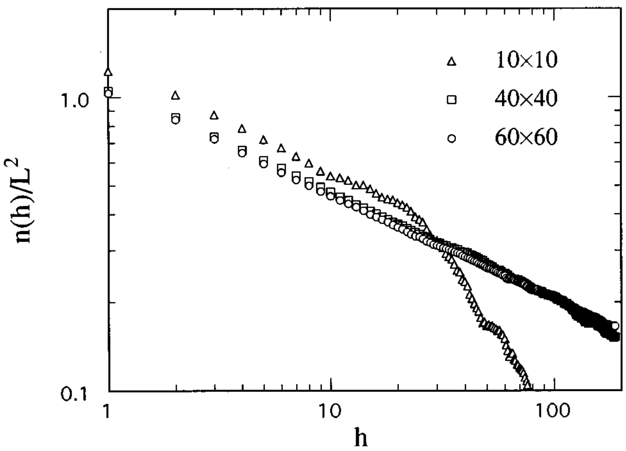

FIG. 2. Log-log plot of the distribution $n(h)$ of the domain walls in a $d=2+1$ half space. Various system sizes are illustrated: 10×10, 40×40, and 60×60.

B domain walls at a height h scales as $n(h)\sim h^{-\chi}$, with $\chi=\frac{2}{3}$ for $L\rightarrow+\infty$ [6] such that the internal pattern of domains presents power-law correlations. The internal pattern is then expected to be scale invariant and the size distribution of A and B domains is expected to be a simple power law. In so doing, the internal pattern is said to be *critical* [6]. This emergence of criticality for competing species seems to be natural and universal.

We numerically investigated the SMK model in $2+1$ and $3+1$ dimensions. Figure 2 presents the decay of $n(h)$ for $d=2+1$ for seed sizes up to 60×60. A *power-law* decay of $n(h)$ was found for large system size, expressing the presence of strong finite-size effects in this geometry. The exponent χ was found to be $\chi=0.35\pm 0.03$, close to $\frac{1}{3}$. For $d=3+1$, seed sizes up to $40\times 40\times 40$ have been simulated and the exponent χ is numerically estimated to be $\chi=0.22\pm 0.06$, close to $\frac{1}{4}$. The values of the exponent χ are listed in Table I.

Now let us consider the growth in a different geometry, i.e., on a d-dimensional hypercubic lattice starting from a central site. Initially, the minimal double seed configuration AB is considered on the central sites of the lattice. The growth consists in selecting at random one occupied site on the cluster surface and in gluing a new particle of the same state on one of its empty neighboring sites. This selection-gluing process is repeated a desired number of times. The process is equivalent to the SMK model, but the geometry is now slightly different: the macroscopic growth is radial and the clusters are rounded.

Figure 3 presents the substructure of a cluster after

TABLE I. Numerical values of the critical exponents χ and α estimated on various d-dimensional hypercubic lattices. The last column gives an estimation of the exponent α through the scaling relationship of Eq. (3).

d	χ	α	$(d-2)(1+\chi)$
2	0.67 ± 0.01	0	0
3	0.35 ± 0.03	1.33 ± 0.01	1.35 ± 0.03
4	0.22 ± 0.06	2.48 ± 0.04	2.44 ± 0.12

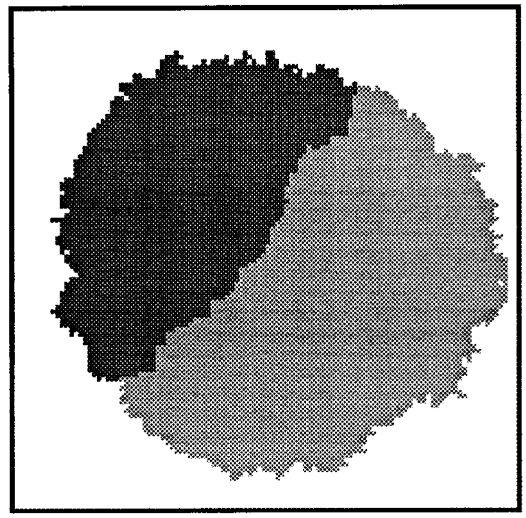

FIG. 3. Two-component cluster grown on the square lattice. Each color represents a species. Note the conservation of the number of domains during the growth.

$N=10\,000$ particles have been glued: each color represents a particle species. Contrary to the confined half space of the SMK model, the competition does not lead asymptotically to the growth of a single domain, but the number of domains seems to be conserved during the cluster growth. This is verified through measuring the number $n(r)$ of domain walls as a function of the distance r from the central lattice site. Clusters of mass up to 2^{23}, 2^{21}, and 2^{20} particles were simulated for $d=2$, 3, and 4, respectively. We found numerically that $n(r)$ scales as a power law $n(r)\sim r^{\alpha}$. The exponent α is thus the counterpart of the exponent χ, but in the radial growth case.

The exponent α was found to be zero on the square lattice ($d=2$), as shown in Fig. 4. Moreover, α is *strictly positive* in three ($\alpha=1.33\pm 0.01$) and four dimensions ($\alpha=2.48\pm 0.04$). This is also illustrated in Fig. 4. One should note that the central seed growth model gives more accurate exponent values than in the restricted growth case since finite-size effects are less important in the former geometry.

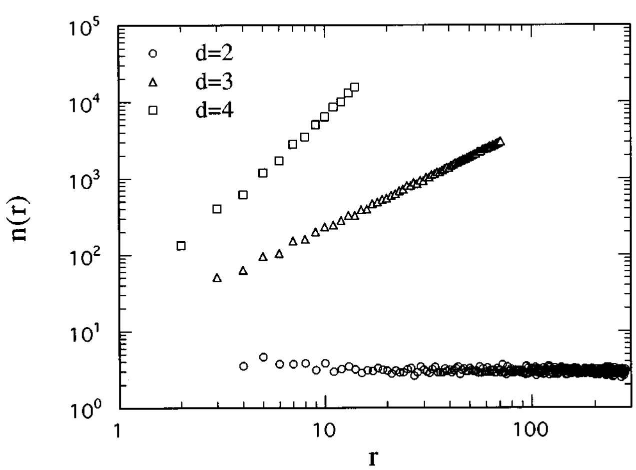

FIG. 4. log-log plot of the radial distribution $n(r)$ of domain walls in, respectively, $d=2$, 3, and 4.

Because new domains cannot spontaneously nucleate on the cluster surface, a positive or zero value of α means that the number of domains is strictly constant during the cluster growth. However, the positive value of α in three and four dimensions means that the domain walls are fractal with a fractal dimension $d_f = \alpha + 1$.

Strictly speaking, the growth of domains in this open space geometry is *noncritical* since the size of growth events (i.e., domain sizes) is not distributed as a power law. However, the domain walls are fractal-like objects, resulting from the motion of the domain walls. It seems that these basic two-component spreading phenomena are not universal in the strict physical sense and depend on the considered space availability. This unexpected nonuniversality was also observed for the roughness exponent of interfaces between Eden clusters [7] in various confined geometries.

One can explain the effect of the geometry on the criticality by describing the formation of internal structures. One may, e.g., assimilate the growth of the diffusion of ''domain-wall particles'' on the cluster surface. When two domain-wall particles meet, they annihilate. The latter event corresponds to the coalescence of two domains. For the SMK model, one can write down a generalized diffusion-annihilation equation [6]

$$\frac{d\rho}{dh} \sim -\rho^{1+1/\chi}, \tag{1}$$

where the density of wall particles is simply $\rho(h) = n(h)/L$. Integrating this equation, one finds easily the power law $n(h) \sim h^{-\chi}$, in agreement with numerical results for the $L \rightarrow +\infty$ limit.

However, one should remark that for the central seed case, the diffusion-annihilation equation should take into account the fact that the average interparticle distance increases with a positive variation of r. Because of this surface dilation, the decreasing rate of the density of wall particles ρ is proportional to $d(1/S)/dr$, where S is the cluster surface $S \sim r^{d-1}$. This argument implies that

$$\frac{d\rho}{dr} \sim -\frac{\rho^{1+1/\chi}}{r^d}, \tag{2}$$

where $\rho(r) = n(r)/r^{d-1}$. Assuming that the number of domain-wall particles scales as $n(r) \sim r^{\alpha}$, Eq. (2) leads to a relationship

$$\alpha = (d-2)(1+\chi) \tag{3}$$

between both exponents α and χ and the space dimensionality d. As shown in Table I, the relationship (3) is in very good agreement with the numerical estimations of both exponents. One should note that $\chi \geqslant 0$ since the number of domains cannot increase. This imposes that $\alpha \geqslant d-2$.

It should be pointed out that the decay of the number of domains in the striplike geometry can be related to the surface fluctuations through the dynamical exponent z [8]. For the Eden growth, the surface fluctuations are governed by the Kardar-Parisi-Zhang (KPZ) equation [9]. Earlier studies in a striplike geometry [8] have shown that the number $N(h)$ of domains goes as

$$N(h) \sim h^{-(d-1)/z} \tag{4}$$

in d-bulk dimension. For $d=2$, the number of domains at height h is equivalent to the number of domain wall particles $n(h)$ such that $\chi = 1/z$. Since $z = \frac{3}{2}$ for $d=2$ [9], this gives $\chi = \frac{2}{3}$, in agreement with our findings. Thus our findings are consistent with the usual exponents for surface fluctuations.

However, for $d>2$, the Eden model does not reach the asymptotic regime of the KPZ equation for accessible length and time scales [10]. For $d>2$, we found herein that $\chi \approx 1/d$.

We conclude from these results that the geometry of a two-component propagation (round or confined in a half space) influences the binary species critical growth behavior. The criticality of two-component spreading seems to be not universal because the geometry affects the criticality. However, a scaling relationship is found between the exponents χ and α describing the domain-wall distributions in the different geometries and the space dimension d. This work enhances the interest of the study of multicomponent spreading phenomena [5] and opens alternative perspectives. Further developments concern, e.g., the case of more than two kinds of particles.

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